Project name: School Project Prioritization and Implementation

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The VIKOR technique was first introduced by Opricovic in 1998 in order to solve multi-criteria decision making (MCDM) problems and obtain the best compromise solution. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. The main objective of the VIKOR method is to choose a solution that is closest to the ideal level in each criterion such that the alternatives are based on the particular measure of ‘‘closeness’’ to the ‘‘ideal’’ solution.

In this study there are 5 criteria and 5 alternatives that are ranked based on VIKOR method. The table below shows the type of criterion and weight assigned to each criterion.

**Characteristics of Criteria**

|  |  |  |  |
| --- | --- | --- | --- |
|  | name | type | weight |
| 1 | criterion1 | + | 0.2 |
| 2 | criterion2 | + | 0.2 |
| 3 | criterion3 | - | 0.2 |
| 4 | criterion4 | + | 0.2 |
| 5 | criterion5 | + | 0.2 |

The following table shows the decision matrix.

**Decision Matrix**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | criterion1 | criterion2 | criterion3 | criterion4 | criterion5 |
| alternative1 | 2 | 5.6 | 10.7 | 3 | 1 |
| alternative2 | 3 | 6.4 | 8.3 | 5 | 2 |
| alternative3 | 5 | 2.32 | 3.3 | 1 | 5 |
| alternative4 | 2 | 14.6 | 10 | 3 | 1 |
| alternative5 | 3 | 10.8 | 6.8 | 2 | 3 |

The VIKOR method involves the following steps:

**The Steps of the VIKOR Method**

**STEP 1: Normalize the decision matrix**

The following formula can be used to normalize.

$$f\_{ij}\left(x\right)=\frac{x\_{ij}}{\sqrt{\sum\_{i=1}^{m}x\_{ij}^{2}}} i=1,…, m ;j=1, … , n$$

The table below shows the normalized decision matrix.

**Normalized Decision Matrix**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | criterion1 | criterion2 | criterion3 | criterion4 | criterion5 |
| alternative1 | 0.28 | 0.277 | 0.58 | 0.433 | 0.158 |
| alternative2 | 0.42 | 0.317 | 0.45 | 0.722 | 0.316 |
| alternative3 | 0.7 | 0.115 | 0.179 | 0.144 | 0.791 |
| alternative4 | 0.28 | 0.723 | 0.542 | 0.433 | 0.158 |
| alternative5 | 0.42 | 0.535 | 0.369 | 0.289 | 0.474 |

**STEP 2: Determine the best** $f \_{i}^{\*}$**and worst** $f \_{i}^{-}$**benefits of each criterion**

The best and worse benefits can be determined by the following formula:

If the criterion is positive, then

$f\_{j}^{\*}=Max\_{i}f\_{ij} , f\_{j}^{-}=Min\_{i}f\_{ij} ; j=1,2,…,n$

If the criterion is negative, then

$$f\_{j}^{\*}=Min\_{i}f\_{ij} , f\_{j}^{-}=Max\_{i}f\_{ij} ; j=1,2,…,n$$

The positive ideal solution ($f^{\*})$ and negative ideal solution ($f^{-}) $can be expressed as follows:

 $f^{\*}=\left\{f\_{1}^{\*},f\_{2}^{\*},f\_{3}^{\*},…,f\_{n}^{\*}\right\}$

$$f^{-}=\left\{f\_{1}^{-},f\_{2}^{-},f\_{3}^{-},…,f\_{n}^{-}\right\}$$

**STEP 3: Calculate the** $S\_{i}$ **and** $R\_{i}$ **values**

The values $S\_{i}$ and $R\_{i}$, representing the group utility and individual regret, respectively, can be calculated by the formulas below :

$$S\_{i}=\sum\_{j=1}^{n}w\_{j}\frac{(f\_{j}^{\*}-f\_{ij})}{(f\_{j}^{\*}-f\_{j}^{-})} $$

$$R\_{i}=Max\_{j}\left[w\_{j}\frac{\left(f\_{j}^{\*}-f\_{ij}\right)}{\left(f\_{j}^{\*}-f\_{j}^{-}\right)}\right]$$

Where $w\_{j}$ denotes the weight of the criteria.

The following table shows the values $S\_{i}$ and $R\_{i}$.

**The values** $S$ **and** $R$

|  |  |  |
| --- | --- | --- |
|  | R | S |
| alternative1 | 0.2 | 0.847 |
| alternative2 | 0.15 | 0.552 |
| alternative3 | 0.2 | 0.4 |
| alternative4 | 0.2 | 0.681 |
| alternative5 | 0.15 | 0.54 |

**STEP4: Calculate the value** $ Q\_{i}$

The value $Q\_{i}$, representing the VIKOR index for each alternative can be calculated by the following formula:

$$Q\_{i}=γ\frac{(S\_{i}-S^{\*})}{(S^{-}-S^{\*})}+(1-γ)\frac{(R\_{i}-R^{\*})}{(R^{-}-R^{\*})}$$

Where

$$S^{\*}=Min\_{i}\left\{S\_{i}\right\} ; S^{-}=Max\_{i}\left\{S\_{i}\right\} ; R^{\*}=Min\_{i}\left\{R\_{i}\right\} ; R^{-}=Max\_{i}\left\{R\_{i}\right\}$$

And $γ$ is the maximum group utility represented by value 0.5.

**The values** $Q$

|  |  |
| --- | --- |
|  | Q |
| alternative1 | 1 |
| alternative2 | 0.17 |
| alternative3 | 0.5 |
| alternative4 | 0.815 |
| alternative5 | 0.157 |

**STEP5: Rank the alternatives, sorting by the S, R and Q values**

Alternatives are ranked by sorting the S, R, and Q, values in decreasing order such that the best rank is assigned to the alternative with the smallest VIKOR value. The results are three ranking lists.

The following table presents the ranking list for the alternatives based on the S, R, and Q values

**The ranking list for the alternatives**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | R value | Rank in R | S value | Rank in S | Q value | Rank in Q |
| alternative1 | 0.2 | 3 | 0.847 | 5 | 1 | 5 |
| alternative2 | 0.15 | 2 | 0.552 | 3 | 0.17 | 2 |
| alternative3 | 0.2 | 3 | 0.4 | 1 | 0.5 | 3 |
| alternative4 | 0.2 | 3 | 0.681 | 4 | 0.815 | 4 |
| alternative5 | 0.15 | 1 | 0.54 | 2 | 0.157 | 1 |

**STEP 6: Propose a compromise solution**

the alternative ($A^{(1)}$), which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

**Condition 1**. Acceptable advantage: $Q(A^{(2)})-Q(A^{\left(1\right)})\geq 1/(m-1)$ where$A^{(1)}$is the alternative with first position and $A^{(2)}$is the alternative with second position in the ranking list by Q. m is number of alternatives.

**Condition 2**. Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by S or/and R.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

**Solution 1.** Alternatives $A^{\left(1\right)} , A^{\left(2\right)}, …. , A^{\left(M\right)} $if Condition 1 is not satisfied; Alternative $A^{\left(M\right)}$ is determined by $Q\left(A^{\left(M\right)}\right)- Q\left(A^{\left(1\right)}\right) <1/(m-1)$ for maximum M (the positions of these alternatives are ‘‘in closeness’’).

**Solution 2.** Alternatives $A^{\left(1\right)}$ and $A^{\left(2\right)}$ if only condition 2 is not satisfied.

**Solution 3.** Alternative with the minimum Q value will be selected as the best Alternative if both conditions are satisfied.

result of the conditions survey is shown in the following table.

**result of the conditions survey**

|  |  |
| --- | --- |
| Non acceptance | Condition 1 |
| - | Condition 2 |
| Solution 1 | Selected solution |

Therefore, alternative5,alternative2, are selected as the final alternatives.